Shrub Ensembles for Online Classification

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Thirty-Sixth AAAI Conference on Artificial Intelligence 2022

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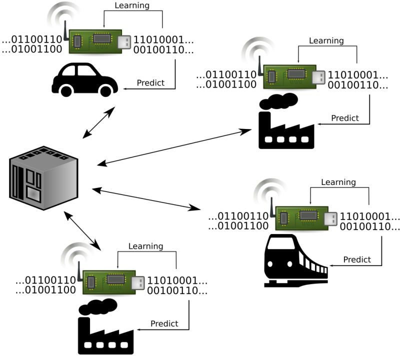
Online Learning on the Edge



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By 2025 $\approx$ 38 Billion IoT Devices world wide[1]

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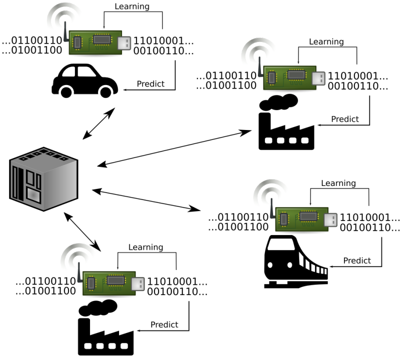

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Clear benefits

+ Privacy and independence of data analysis
+ Reduced communication infrastructure costs
+ Faster response times

[1] https://findstack.com/internet-of-things-statistics/

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Evolving data streams

The algorithm must adapt to changes in the new data distribution and preserve its performance.

Online Decision Tree Ensemble Learning

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+ constant memory consumption
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Can we design simple <u>and</u> small online ensembles?

$$\ldots, (x_0, y_0), \boxed{(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), (x_5, y_5)}, (x_6, ?), (x_7, ?), (x_8, ?) \ldots$$

$$\longrightarrow \text{train} \longrightarrow \text{h(x)} \longrightarrow$$

+ We can use a batch algorithm to train $h(x)$
+ We quickly adapt to concept drift
- No long-term learning possible

Revisiting Sliding Windows

$$\ldots, (x_1, y_1), \boxed{(x_2, y_2), (x_3, y_3), (x_4, y_4), (x_5, y_5), (x_6, y_6)}, (x_7, ?), (x_8, ?), (x_9, ?) \ldots$$

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Hence The two classifiers h_1 and h_2 are redundant. We should use h_1

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Idea Learn an ensemble of local patterns from sliding windows

$$f(x) = \sum_{i=1}^{M} w_i h_i(x)$$

Setting and Notation

Formally Let $\mathcal{H} = \{h_1, h_2, \dots, h_K\}$ be the set of trees learned on local patterns:

$$\arg\min_{w \in \mathbb{R}^K} \sum_{t=1}^{T} \ell\left(f_{S[0:t-1]}(x_t), y_t\right) \text{ s.t. } \|w\|_0 \leq M, w_i \geq 0, \sum_{i=1}^{K} w_i = 1$$

with

- $f_{S[0:t-1]} \colon \mathbb{R}^d \to \mathbb{R}^C$ is the model at time t
- $\ell \colon \mathbb{R}^C \times \mathcal{Y} \to \mathbb{R}_+$ is the loss

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For this talk Focus on the MSE loss

$$\ell(f_{S[0:t-1]}(x_t), y_t) = \frac{1}{C}\left\|f_{S[0:t-1]}(x_t) - y_t\right\|^2$$

Proximal Gradient Descent

$$w \leftarrow \mathcal{P}\left(w - \alpha \nabla_w L_{\mathcal{B}}(w)\right),$$

where

- $\mathcal{B}$ is the current window with $|\mathcal{B}| = B$ examples

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Our paper / Kyrillidis et al. 2013 Details for prox-operator

Putting it all together: Shrub Ensembles (SE)

Idea If a tree is helpful to the ensemble, then it should have a large weight

Algorithm 1: Shrub Ensembles.

$w \leftarrow (0); \mathcal{B} \leftarrow [\,]; \mathcal{H} \leftarrow [\,]$ ▷ Init.
for next item (x, y) **do**
    if $|\mathcal{B}| = B$ **then** ▷ Update batch
        $\mathcal{B}$.`pop_first()`
    $\mathcal{B}$.`append`$((x, y))$
    $h_{new} \leftarrow \texttt{train}(\mathcal{B})$ ▷ Add new classifier
    $\mathcal{H}$.`append`(h_{new})
    $w \leftarrow (w_1, \ldots, w_M, 0)$ ▷ Initialize weight
    $w \leftarrow w - \alpha \nabla_w L_{\mathcal{B}}(w)$ ▷ Gradient step
    $w, \mathcal{H} \leftarrow \texttt{sorted}(w, \mathcal{H})$ ▷ Sort decreasing order
    $w \leftarrow \mathcal{P}(w)$ ▷ Project on feasible set
    $w, \mathcal{H} \leftarrow \texttt{prune}(w, \mathcal{H})$ ▷ Remove zero weights

Theoretical Insights

Runtime $\mathcal{O}\left(dB^2 \log B + \log M\right)$ per example with d features

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window

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Behaviour Let $m \leq M$ be the number of models in the ensemble and let $\forall j = 1, \ldots, m \colon h_j(x_B) \neq y_B$. If SE trains fully-grown trees such that $\forall i = 1, \ldots, B \colon h(x_i) = y_i$ it holds for $\alpha > \frac{BC}{4m}$ that:

- (1) If $m < M$, then h is added to the ensemble
- (2) If $m = M$, then h replaces the tree with the smallest weight from the ensemble

Experimental Insights: Quantitative Analysis

Goal Compare Accuracy-Memory Trade-off of different configurations

1) Plot Pareto Front of best performing configurations against accuracy
2) Compute Area Under the Pareto Front to measure accuracy-memory trade-off
3) Rank each method according to its trade-off and plot a CD-diagram

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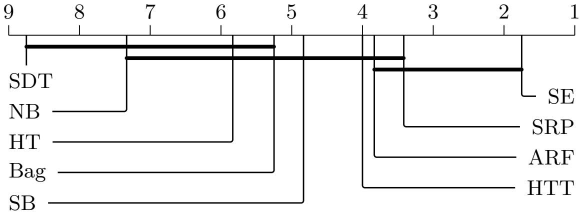
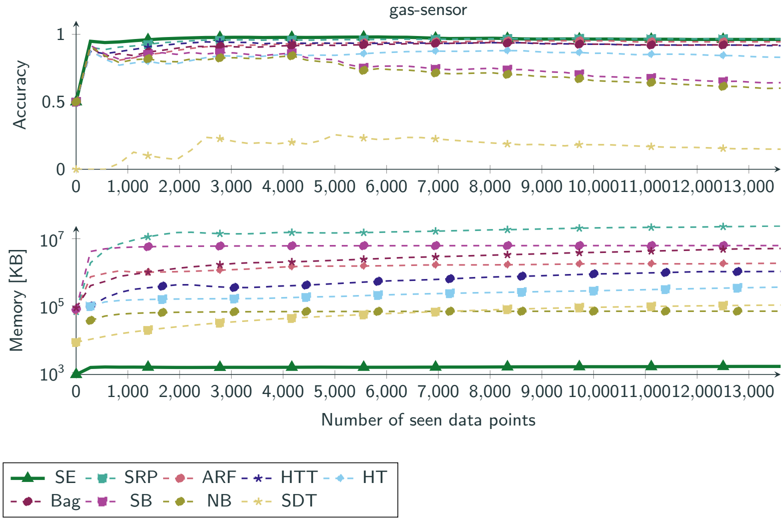
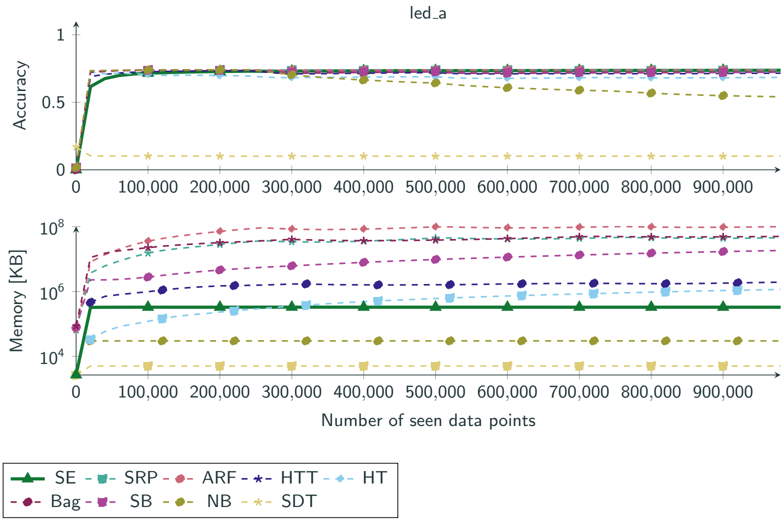


- SE: Shrub Ensembles
- SRP: Streaming Random Patches
- ARP: Adaptive Random Forest
- HTT: HoeffdingAnyTree
- SB: (Online) Smooth Boost
- Bag: (Online) Bagging
- HT: HoeffdingTrees
- NB: (Online) NaiveBayes
- SDT: Soft Decision Trees

Experimental Insights: Qualitative Analysis (1)



Experimental Insights: Qualitative Analysis (2)



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Current approaches Unbounded memory or costly gradients

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Current approaches Unbounded memory or costly gradients

Shrub Ensembles Bounded memory and simple gradients

- Train small trees ($\rightarrow$ Shrubs) on sliding window
- Maintain tree weights via proximal gradient descent
- Prune unimportant trees

Results Better accuracy-memory trade-off than existing methods!